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Λ-CDM Universe: A Phenomenological Approach With Many Possibilities

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A time-dependent phenomenological model of Λ, viz., $\dot{\Lambda} \sim H^3$ is selected to investigate the Λ-CDM cosmology. Time-dependent form of the equation of state parameter ω is derived and it has been possible to obtain the sought for flip of sign of the deceleration parameter q. Present age of the Universe, calculated for some specific values of the parameters agrees very well with the observational data.

Keywords: dark energy, variable Λ, Λ-CDM cosmology

1. Introduction

Modern cosmological research rests heavily on observational data. Any theoretical model should be corroborated with observation for understanding the viability of that model. Present cosmological picture, emerging out of this theory-observation combination, reveals that, the total energy-density of the Universe is dominated by two dark components, viz., dark matter and dark energy. Observational evidence from various independent sources including SN Ia[1,2,3,4,5,6,7,8] suggest that the cosmic expansion is speeding up, i.e. accelerating. This acceleration is supposed to be caused by a yet unknown exotic energy, termed as dark energy. A special feature of dark energy is that it exerts negative pressure which acts as a repulsive force initiating the observed acceleration. Not only that, it is now well-accepted that about two-third of the total energy-density comes in terms of dark energy while the remaining one-third is contributed by matter, both visible and dark[9].

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Now, dark matter played a significant role in the early Universe during structure formation because it clumps in sub-megaparsec scales. But, the exact composition of dark matter is still unknown. Since small density perturbation ($\delta\rho/\rho \sim 10^{-5}$ at $z \simeq 1100$) measured by COBE and CMB experiments[9] rule out baryonic dark matter and hot dark matter like light neutrinos do not support hierarchical structure formation [10,11,4,12], so most of the dark matter must be cold and non-baryonic. On the other hand, clustering of cold dark matter on small scale[9] supports the hierarchical structure formation. Moreover, after introduction of the idea of accelerating Universe, the previous Standard Cold Dark Matter (SCDM) models have fallen out of grace[13,14] and is replaced by Λ-CDM or LCDM model for including dark energy as a part of the total energy density of the Universe. Λ-CDM model is found to be in nice agreement with various sets of observations[15]. An advantage of Λ-CDM model is that it assumes a nearly scale-invariant primordial perturbations and a Universe with no spatial curvature. These were predicted by inflationary scenario[16,17,18,19,20].

Now, a problem with Λ-CDM model is that the acceleration of the Universe cannot be a permanent feature starting from the Big-Bang. Because, an accelerating Universe is not favorable for structure formation. This problem can be removed if one assumes that the acceleration of the Universe is a recent phenomena. In fact, some recent works[21,22] show that the present accelerating phase was preceded by a decelerating one and observational evidence[23] also supports this idea. The present work is done with this background in mind.

Phenomenological approach is one of the several ways of searching such dark energy. In a recent work[24], the equivalence of three phenomenological variable Λ models have been shown. The behaviour of the same three forms of Λ have been studied when both G and Λ vary[25]. But, in both those works the equation of state parameter ω was considered as a constant because, due to inability of current observational data in separating a time-varying ω from a constant one[26,27], in most of the cases a constant value of ω is used. However, ω, in general, is a function of time[28,29,30]. It has already been commented by Ray et al.[24] that for a more accurate result, an investigation regarding time evolution of ω may be taken up for searching better physical features. In fact, the Statefinder diagnostic, used for distinguishing various dark energy models, can be applied if the equation of state of scalar potential has a direct relationship with the Hubble parameter and its derivative[31,32,9].

With those features of q and ω in mind, an investigation about the Λ-CDM Universe is done by selecting a specific time-dependent form of Λ, viz., $\dot{\Lambda} \sim H^3$. This particular time-varying Λ model was studied by Reuter and Wetterich[33] for finding a mechanism which would explain the present small value of Λ as a result of the cosmic evolution. In the present work, the same Λ model is used for investigating a time evolving equation of state parameter ω along with a possible signature flip of the deceleration parameter q. This change of sign is very important for Λ-CDM cosmology.

2. Field Equations

The Einstein field equations are given by

$$R^{ij} - \frac{1}{2}Rg^{ij} = -8\pi G\left[T^{ij} - \frac{\Lambda}{8\pi G}g^{ij}\right] \tag{1}$$

where the cosmological term Λ is time-dependent, i.e. $\Lambda = \Lambda(t)$ and c, the velocity of light in vacuum, is assumed to be unity.

Let us consider the Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{2}$$

where k, the curvature constant, assumes the values -1, 0 and $+1$ for open, flat and closed models of the Universe respectively and $a = a(t)$ is the scale factor. For the spherically symmetric metric (2), field equations (1) yield Friedmann and Raychaudhuri equations respectively given by

$$3H^2 + \frac{3k}{a^2} = 8\pi G\rho + \Lambda, \tag{3}$$

$$3H^2 + 3\dot{H} = -4\pi G(\rho + 3p) + \Lambda \tag{4}$$

where G, ρ and p are the gravitational constant, matter energy density and pressure respectively and the Hubble parameter H is related to the scale factor by $H = \dot{a}/a$. In the present work, G is assumed to be constant. The generalized energy conservation law for variable G and Λ is derived by Shapiro et al.[34] using Renormalization Group Theory and also by Vereshchagin and Yegorian[35] using a formula of Gurzadyan and Xue[36]. Vereshchagin and Yegorian[37] have presented a phase portrait analysis of the cosmological models relying on the Gurzadyan-Xue type dark energy formula as mentioned above. A novel interpretation of the physical nature of dark energy and description of an internally consistent solution for the behavior of dark energy as a function of redshift are provided by Djorgovski and Gurzadyan[38] based on the vacuum fluctuations model by Gurzadyan and Xue[36].

The conservation equation for variable Λ and constant G is a byproduct of the generalized conservation law and is given by

$$\dot{\rho} + 3(p+\rho)H = -\frac{\dot{\Lambda}}{8\pi G}. \tag{5}$$

Let us consider a relationship between the pressure and density of the physical system in the form of the following barotropic equation of state

$$p = \omega\rho \tag{6}$$

where ω is the barotropic index which has been considered here as time-dependent.

Using equation (6) we get from (5)

$$8\pi G\dot{\rho} + \dot{\Lambda} = -24\pi G(1+\omega)\rho H. \tag{7}$$

Differentiating (3) with respect to t we get for a flat Universe ($k = 0$)

$$-4\pi G\rho = \frac{\dot{H}}{1+\omega}. \tag{8}$$

As already mentioned in the introductory part, equivalence of three phenomenological Λ-models (viz., $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$) have been studied in detail. So, similar type of variable-Λ model may be investigated for a deeper understanding of both the accelerating and decelerating phases of the Universe. Let us, therefore, use the *ansatz*, $\dot{\Lambda} \propto H^3$, so that

$$\dot{\Lambda} = AH^3 \tag{9}$$

where A is a proportional constant.

Using equations (6), (8) and (9) we get from (4)

$$\frac{2}{(1+\omega)H^3}\frac{d^2H}{dt^2} + \frac{6}{H^2}\frac{dH}{dt} = A. \tag{10}$$

If we put $dH/dt = P$, then equation (10) reduces to

$$\frac{dP}{dH} + 3(1+\omega)H = \frac{A(1+\omega)H^3}{2P}. \tag{11}$$

To arrive at fruitful conclusions, let us now solve equation (11) under some specific assumptions.

3. Solutions

3.1. $A = 0$

$A = 0$ implies via equation (9), $\Lambda = constant$. In this case equation (11) reduces to

$$\frac{dP}{dH} + 3(1+\omega)H = 0. \tag{12}$$

Solving equation (12) for $a(t)$, $\rho(t)$ and $H(t)$ we get

$$a(t) = C_1 t^{2/3(1+\omega)}, \tag{13}$$

$$H(t) = \frac{2}{3(1+\omega)}\frac{1}{t}, \tag{14}$$

$$\rho(t) = \frac{1}{6\pi G(1+\omega)^2}\frac{1}{t^2} \tag{15}$$

where C_1 is a constant.

It may be mentioned here that the above expressions for $a(t)$, $H(t)$ and $\rho(t)$ can be recovered from the corresponding expressions of Ray et al.[24] for $\alpha = 0$, i.e. $\Lambda = 0$ where α is a parameter for the model $\Lambda \sim H^2$ considered there. This means that the results (13), (14) and (15) can be obtained either for constant Λ (as in the present case) or for null Λ as in the case of Ray et al.[24]. The essence of this is that,

a null Λ or constant Λ will provide equivalent result. It may also be mentioned here that by abandoning Λ, Einstein obtained the expanding Universe while the same expanding Universe was obtained by de Sitter for constant Λ.

Again, using equation (14) we can find the expression for the deceleration parameter q as

$$q = -\left(1 + \frac{\dot{H}}{H^2}\right) = \left(\frac{1+3\omega}{2}\right). \tag{16}$$

From equation (16) we find that for an accelerating Universe, $\omega < -1/3$. But, from equation (13)-(15) it is clear that ω cannot be equal to -1. Moreover, the present value of q lies near -0.5[39] which can be obtained from equation (16) by putting a value of ω which is equal to $-2/3$. The sought for signature flipping of q can be obtained from equation (16) if one considers ω as time-dependent.

If H_0 and t_0 be the present values of H and t, then from equation (14) we can write,

$$t_0 = \frac{2}{3(1+\omega)H_0}. \tag{17}$$

Putting $\omega = -1/3$ in equation (17) and assuming $H_0 = 72$ kms^{-1}Mpc^{-1} we find that the present age of the Universe comes out as 13.58 Gyr. which fits very well within the ranges provided by various sources (for a list of data provided by various sources one may consult Ray et al. [24]. In this context it may be mentioned that for stiff-fluid ($\omega = 1$) Ray et al.[40] obtained the present age of the Universe as 13.79 Gyr. under the ansatz $\Lambda \sim H^2$.

3.2. $1 + \omega = -2P/H$

By the use of the above substitution Eq. (11) becomes

$$\frac{dP}{dH} - 6P = -AH^2. \tag{18}$$

Solving equation (18) we get

$$a(t) = C_2 e^{-t/6}(secBt)^{1/6B}, \tag{19}$$

$$H(t) = \frac{1}{6}(tanBt - 1), \tag{20}$$

$$\Lambda(t) = \frac{B}{6}\left[\frac{1}{2B}tan^2Bt + \frac{2}{B}log(secBt) - \frac{3}{B}tanBt + 2t\right], \tag{21}$$

$$\rho(t) = \frac{1}{48\pi G}(tanBt - 1), \tag{22}$$

$$\omega(t) = -\left[1 + \frac{2Bsec^2Bt}{(tanBt - 1)}\right] \tag{23}$$

where C_2 is a constant and $B = A/36$.

For physically valid H, $tanBt > 1$. Then Eq. (23) implies that for a positive B, ω must be less than -1 as in the case of phantom energy. On the other hand if $B < 0$, then ω can be greater than -1 as well.

Similar type of trigonometric solutions have been obtained by Mukhopadhyay and Ray[41] for polytropic equation of state with constant ω in non-dust case under the *ansatz* $\Lambda \sim \ddot{a}/a$. Simple trigonometric solution for the scale factor was also obtained by Banerjee and Das[42] in scalar field model. But, they obtained their solution by making a special assumption on the deceleration parameter while the present solution is a result of a supposition on the equation of state parameter ω.

Again, using equation (20) we get

$$q = -\left[1 + \frac{6Bsec^2Bt}{(tanBt - 1)^2}\right]. \tag{24}$$

From equation (24) we find that, a signature flipping of q is possible if $B < 0$. So, the merit of this case lies the fact that the same change of sign of q can be obtained here by using a time-dependent form of ω and not making any special assumption on q directly as was done by Banerjee and Das[42]. This once again shows that the equation of state parameter is a key ingredient of cosmic evolution.

3.3. $1 + \omega = -P/3H^2$

With the above assumption, Eq. (11) becomes

$$\frac{dP}{dH} - \frac{P}{H} = -\frac{A}{6}H. \tag{25}$$

Solving equation (25) we get our solution set as

$$a(t) = C_4 t^{6/A}, \tag{26}$$

$$H(t) = \frac{6}{At}, \tag{27}$$

$$\rho(t) = \frac{27}{\pi G A^2}\frac{1}{t^2}, \tag{28}$$

$$\Lambda(t) = -\frac{108}{A^2}\frac{1}{t^2}, \tag{29}$$

$$\omega(t) = \frac{A}{18} - 1 \tag{30}$$

where C_4 is an integration constant.

Thus, we find that in this case the scale factor admits a power law solution, H varies inversely as t and ρ as well as Λ follow the well known inverse square law with t. This type of solution was obtained by Ray et al.[24] for $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$ models. For physical validity $A > 0$. But, in this case $\Lambda < 0$ for real

A and hence represents an attractive force. However, Λ can be a repulsive force as well if A is a complex number. Now, a complex A means a complex scale factor. So, this particular case can be thought of as a phenomenological version of spintessence model of Banerjee and Das[43] where a complex scalar field of the form $\phi = e^{i\omega t}$ is used to search for the cosmic acceleration. But, in that case ω is a constant. Also, for $A = 6$ if we take the present value of the Hubble parameter as $H_0 = 72$ $\mathrm{kms^{-1}Mpc^{-1}}$ then, from equation (27) the present age of the Universe comes out as 13.58 Gyr. which agrees very well with the estimated value[24]. Now, for $A = 6$ we have $\omega = -2/3$ and the scale factor grows linearly with time. Again, in this case

$$q = -\left[1 + \frac{A}{6}\right]. \tag{31}$$

Equation (31) shows that for a positive A, the Universe expands with a constant acceleration. For $A = 6$ the amount of acceleration is -2. So, in this case of the phenomenological model $\dot{\Lambda} \sim H^3$, the deceleration parameter q does not show any change in sign during cosmic evolution.

4. Discussions

The main objectives of the present work were to search for a signature flip of q and to find time-dependent expression for the equation of state parameter. In that respect, this work in general has fulfilled its goal. By selecting a time dependent form of the cosmological parameter Λ, through some analytical study, it has been possible to show that a change in sign of the deceleration parameter can be achieved under some special assumptions (Sec. 3.2). It has also been possible to derive time dependent expressions for the equation of state parameter ω. It is found that ω can be less than -1 as well which is compatible with SN Ia data[3] and SN Ia data with CMB anisotropy and galaxy-cluster statistics[15].

It would be worthwhile to mention here that in the present work the parameter A plays a vital role for studying the cosmic evolution of various phases of the Universe. For instance, a null A presents us a case of constant Λ (Sec. 3.1) whereas positive and negative A show the possibility of $\omega < -1$ (phantom energy) or $\omega > -1$ respectively (Sec. 3.2). The well deserved signature flipping of q is also possible for $A < 0$ (Sec. 3.2). A possibility of a complex A is provided in Sec. 3.3 which has similarity with the work of Banerjee and Das[43]. For a specific value of A the age of the Universe and the value of q are calculated also (Sec. 3.3). It is interesting to note that similar type of case study for the cosmic evolution has been done by Khachatryan[44] using a parameter b for null, positive and negative values of it. So, whether there exists any internal physical relationship between the present work and that of Khachatryan[44] may be a subject matter of future investigation.

Determination of the present value of the Hubble parameter through analysis of CMB data from WMAP and HST Key Project suggests that value of the equation of state parameter for dark energy models should be less than -0.5 at the 95% confidence level[4]. So, in some cases a Big Rip may not be impossible. For Λ-CDM

models, the Statefinder diagnostic satisfies the condition $\frac{d\ddot{a}}{dt}/aH^3 = 1$. Since $\frac{d\ddot{a}}{dt}/a$ can be expressed in terms of H, $\dot{H}$ and $\ddot{H}$, so it is easy to verify that first (Sec. 3.1) and third (Sec. 3.3) cases satisfy the above condition prescribed by the Statefinder diagnostic for $\omega = 0$ and $A = 9$ respectively.

However, the Λ-CDM Universe with $\omega = -1$, where the sine hyperbolic form of the scale factor can reflect both matter dominated past and accelerated expansion in future[45], can not be achieved through this model. Equation (11) shows that for $\omega = -1$, H grows linearly with time which does not fit with the present cosmological scenario. However, through the present model, it has been possible to provide some interesting situations which were obtained earlier by different researchers and are already discussed in respective Sections. Some awkward cases, such as constant energy density (which can be obtained by putting $1+\omega = 2P$ in equation (11)) can be found in the work of Ray[46] in relation to electromagnetic mass in $n+2$ dimensional space-time. Some other works[47,48,49,50,51,52] also admit constant matter distribution in their solutions. Finally, it should be mentioned that the present work is done making Λ variable and keeping G constant. So, it may be an interesting study when the present model is combined with a variable G and obey the generalized energy conservation law derived by Shapiro et al.[34] and Vereshchagin and Yegorian[35]. That can be a subject matter of our future investigation.

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